Nonequilibrium roughening transition in a volume conserving system

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We introduce a simple volume conserving stochastic model undergoing a nonequilibrium roughening transition (NRT) in $1+1$ dimensions. In our model, there is no deposition and evaporation of a particle breaking the volume conserving condition. The degree of roughness of the fluctuating interface in our model is determined by whether or not the hopping of a particle depends on the local slope of the interface. The hopping process of a particle is controlled by the probability $0 \leqslant p \leqslant 1$. For $p < 1/2$, a moving particle tends to hop in the downhill direction of the local slope of the interface, and so the interface is in a smooth phase with a zero roughness exponent. For $p > 1/2$, a particle tends to hop in the uphill direction, and so the interface cannot reach a saturated phase. When $p = 1/2$, the hopping of a particle does not depend on the local slope of the interface. Then the interface can reach a saturated phase. The saturated interface at $p = 1/2$ is in a rough phase with a nonzero roughness exponent. Our model, therefore, exhibits the NRT at the critical parameter $p_c = 1/2$.

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The nonequilibrium roughening transition (NRT) in $(1+1)$-dimensional systems has recently attracted much interest [1–5]. The NRT in $1+1$ dimensions is a very interesting phenomenon because an interface under thermal equilibrium cannot undergo a roughening transition in $1+1$ dimensions. An interface under thermal equilibrium is always in a rough state with a diverging width in $1+1$ dimensions. In higher dimensions, an interface under thermal equilibrium can exhibit the roughening transition from a smooth phase to a rough one with diverging width or vice versa at some critical temperature. However, an interface far from equilibrium can exhibit the NRT even in $1+1$ dimensions, although there are few examples [1–5].

Recently Alon *et al.* [1] introduced a simple growth model exhibiting the NRT in $1+1$ dimensions. In the model, the dynamics is defined as follows. First select a site i randomly. At the site i, a particle is deposited $h_i \rightarrow h_i+1$ with probability p or evaporated at the edge of a step $h_i \rightarrow \min(h_i, h_{i+1})$ with probability $(1-p)/2$ or $h_i \rightarrow \min(h_i, h_{i-1})$ with probability $(1-p)/2$. In the restricted solid-on-sold (RSOS) version of the model, the above processes occur only if the constraint $|h_i - h_{i\pm 1}| \leqslant 1$ is respected. In the unrestricted model, the above processes are always carried out without any restricting condition. Alon *et al.* measured the roughness exponent ζ in their model by changing p. They found that ζ is zero when $p < p_c$ $[=0.232\,67(3)]$ and positive when $p > p_c$ for the unrestricted model. They also found that ζ is zero when $p < p_c$ $[=0.1889(1)]$ and positive when $p > p_c$ for the RSOS model. Therefore the model shows a NRT from a smooth phase with $\zeta = 0$ to a rough one with $\zeta > 0$ at the critical value p_c in $1+1$ dimensions. Alon *et al.* mentioned in their paper that when evaporation of a particle in the middle of a plateau is allowed the interface formed by their model is always rough and no smooth phase exists. In a volume conserving system, however, the interface can exhibit a NRT in $1+1$ dimensions even when the movement of a particle in the middle of a plateau is allowed.

In this paper, we introduce a simple stochastic model satisfying the volume conserving condition. By carrying out a Monte Carlo simulation of the model, we show that the volume conserving model exhibits the NRT in $1+1$ dimensions even when the movement of a particle in the middle of a plateau is allowed. The dynamic rule of our model is as follows. Each time a site i is selected randomly. It is checked whether the height at any neighboring site is greater than h_i by at least one lattice spacing. If so, the particle at site i is regarded as immobile and a new site is chosen. If not, the particle at site i is moved to the nearest neighbor site according to the following processes. If $(h_{i-1} - h_{i+1}) < 0$, the particle can hop to site $i+1$ with probability p or hop to site $i-1$ with probability $1-p$. If $(h_{i-1} - h_{i+1}) > 0$, the particle can hop to site $i-1$ with probability p or hop to site $i+1$ with probability $1-p$. If $(h_{i-1} - h_{i+1}) = 0$, the particle can move to a randomly chosen one of its nearest neighbor sites. Note that dynamics can occur in our model even when $p = 0$. Our model is a simple discrete model for interface reconstruction without the deposition of particles. When p is very small, a particle at a randomly selected site moves in a downhill direction with great probability when there is a local slope at the selected site i. Only when a particle is in the middle of a plateau in our model can it move to one of the nearest neighbor sites regardless of p. In that case, the height of the randomly selected particle decreases by 1 and the height of one of its neighbor sites increases by 1. This movement of a selected particle on the plateau makes the interface become rougher even for small p. However, this movement of a particle occurs rarely in comparison with the downhill movement for small p after the interface goes to a saturated state. The downhill movement of a particle makes the interface become smooth. Therefore, the interface formed by our model is in a smooth phase when p is small. As p increases from a small value, many more particles tend to move to a nearest neighbor site with greater height and the interface becomes rougher. When $p = 1/2$, the dynamics of our model is the same as that of the model introduced by Krug [6]. The

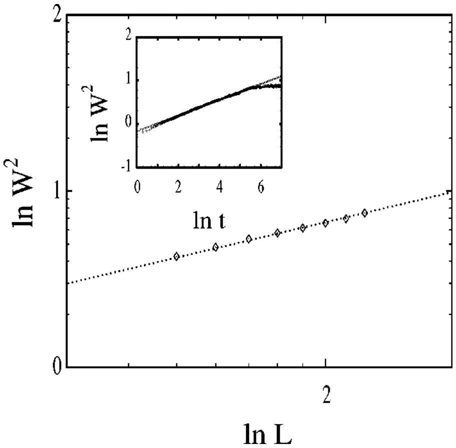

FIG. 1. Plot of $W^2(L,t)$ vs system size L at $p=0.5$ on a double logarithmic scale for system sizes $L=50$, 60, 70, 80, 90, 100, 110, and 120. The line obtained from the least squares fit has the slope $2\zeta=2/3$. Inset: Plot of $W^2(L,t)$ vs time t for the system size $L=120$. The slope of the line is 2/11.

dynamics of the Krug model is well described by the conserved Kardar-Parisi-Zhang (CKPZ) equation [6],

$$\frac{\partial h(x,t)}{\partial t}=-K\nabla^4 h+\lambda\nabla^2(\nabla h)^2+\eta_c(x,t). \qquad (1)$$

Here $h(x,t)$ is the height of the interface at position x and time t. The conserved noise satisfies $\langle\eta_c(x,t)\rangle=0$ and $\langle\eta_c(x,t)\eta_c(x',t')\rangle=-2D\nabla^2\delta^d(x-x')\delta(t-t')$, where d and D are the substrate dimension and a constant, respectively.

The CKPZ equation shows a nontrivial scaling behavior in the interface width. The interface width is defined by $W(L,t)=\langle L^{-d'}\Sigma_i[h_i(t)-\overline{h}(t)]^2\rangle^{1/2}$, which scales as

$$W(L,t)\sim\begin{cases}t^{\zeta/z} & \text{if } t\ll L^z\\ L^\zeta & \text{if } t\gg L^z.\end{cases} \qquad (2)$$

Here $\overline{h}$, L, d', and $h_i(t)$ denote the mean height, system size, substrate dimension, and the height at time t and site i, respectively. ζ, z, and $\beta=\zeta/z$ are called the roughness, dynamic, and growth exponents, respectively. The roughness and growth exponents of the CKPZ equation can be obtained easily by solving the CKPZ equation analytically. The roughness and growth exponents are $\zeta=(2-d)/3$ and $(2-d)/(10+d)$.

We carried out simulations of our model in $1+1$ dimensions for system size $L=40$–120 at $p=1/2$. We could not carry out computer simulations for system size $L>120$ because of the long saturation time. Numerical data were averaged typically over 100 configurations. In order to obtain the growth exponent, we measured the time-dependent behavior of the interface width $W(L,t)$ starting from an initially flat interface. We plotted $W^2(L,t)$ vs time t on a double logarithmic scale in the inset of Fig. 1. The interface width grows with the exponent $\beta\simeq 1/11$. The result agrees with $\beta=1/11$ as expected from the CKPZ equation. Next, in order to obtain the roughness exponent, we plotted the saturated value of $W^2(L,t)$ vs system size L on a double logarithmic scale in Fig. 1. We obtained $\zeta\simeq 1/3$. The result also agrees with $\zeta=1/3$ as expected from the CKPZ equation. Therefore, the saturated interface in our model when $p=1/2$ is rough because $\zeta>0$. From this fact, we can infer that a critical p_c for the transition exists between $p=0$ and $p=1/2$ if a NRT occurs.

Recently, Jung and Kim [7] studied the effect of symmetry on a volume conserving model without deposition and evaporation by using the master equation approach. They found that a Laplacian term is essentially absent from the continuum equation for the dynamics of the model with a symmetrical hopping rate. However, they showed that a Laplacian term can occur in the continuum equation when there is an asymmetrical hopping rate in the stochastic rule of a volume conserving model. They succeeded in deriving a general Langevin-type continuum equation from the growth rule with an asymmetrical hopping rate,

$$\frac{\partial h(x,t)}{\partial t}=\nu\nabla^2 h-K\nabla^4 h+\lambda_1\nabla^2(\nabla h)^2+\lambda_2(\nabla h)^3+\eta_c(x,t). \qquad (3)$$

In our model, symmetrical hopping occurs only when $p=1/2$. We showed via computer simulation that the dynamics of our model with $p=1/2$ is well described by the CKPZ equation without a Laplacian term. It is easy to find by using Jung and Kim's argument [7] that ν has a positive value when $p<1/2$ in our model and ν has a negative value when $p>1/2$. When $\nu<0$, the interface formed by Eq. (3) is well known to be unstable due to the antigravity effect [8]. The width of the interface with $\nu<0$ grows continuously and so does not reach a steady state. That means that the morphology of the interface become rougher continuously as time elapses. We found that the width of the interface formed by our model grows continuously without reaching a steady state for a large system when $p>1/2$. Therefore the growing interface is unstable and very rough when $p>1/2$. When $p<1/2$ (i.e., $\nu>0$), the interface formed by Eq. (3) is stable and in a smooth state because the Laplacian term tends to flatten the fluctuating interface. We found that the roughness exponent ζ is zero at $p=0$. Therefore the critical p_c for the NRT must exist between $0<p\leqslant 1/2$.

We carried out simulations by changing p from $p=1/2$ to $p=0$. We found that the roughness exponent ζ becomes 0 as soon as p becomes smaller than 1/2. We examined the scaling behavior of the saturated interface width in the case of $p=0.499$. The width shows a logarithmic behavior as $W^2\sim(\ln L)^\gamma$, $\gamma=2.10\pm 0.03$, after saturation (see Fig. 2). This means that the roughness exponent ζ is zero. Therefore, we can conclude that our model exhibits the NRT at $p_c=1/2$.

The NRT occurring in a growing interface in $1+1$ dimensions is known to be related to directed percolation (DP) or parity conserving (PC) dynamics [1,2,9–11]. A few growth models exhibiting the NRT in $1+1$ dimensions have been introduced recently. In the models, the DP or PC process emerges at a particular reference height of the interface [1–3]. The reference height of the models is the bottom layer of the interface. The sites where the interface touches the

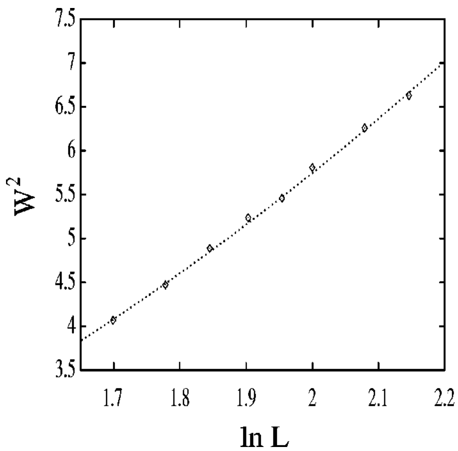

FIG. 2. Plot of $W^2(L,t)$ vs system size L at $p=0.5$ for system sizes $L=50$, 60, 70, 80, 90, 100, 110, and 120. The curved line is $W^2 \sim (\ln L)^{2.10}$.

reference height correspond to the active sites of DP or PC. Therefore, in an active phase of DP or PC, the interface fluctuates close to the reference level so that the interface is smooth. On the other hand, in the inactive phase of DP or PC, the interface detaches from the reference level and evolves into a rough state. Then the interface grows with nonzero velocity and becomes rough. In our model, the NRT is not related to DP or PC. The interface in our model always fluctuates near its average height regardless of the phase, rough or smooth, and the average velocity of the interface is always zero. Therefore, we cannot define the reference height in our model because there exists no inactive phase of DP or PC.

In conclusion, we have introduced a simple volume conserving stochastic model exhibiting a nonequilibrium roughening transition in $1+1$ dimensions. We considered the general Langevin-type continuum equation describing the dynamics of our model for different p. When $p<1/2$, we found that there exists a Laplacian term with a positive coefficient in the general Langevin-type continuum equation for the dynamics of our model. The Laplacian term with positive coefficient tends to flatten the fluctuating interface. We found $\zeta=0$ for $p<1/2$, i.e., the interface is smooth in our model. When $p=1/2$, the dynamics of our model is described well by the CKPZ equation where there is no Laplacian term. The roughness exponent expected from the CKPZ equation is $\zeta=1/3$ in $1+1$ dimensions. Therefore, the interface is not smooth when $p=1/2$. We obtained $\zeta\simeq 1/3$ from simulation of our model. When $p>1/2$, the dynamics of our model is affected by a Laplacian term with a negative coefficient. The interface, then, is always unstable and does not reach a steady state. We also found that the interface is unstable for a large system from simulation of our model when $p>1/2$.

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